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Counterexample to boundary regularity of a strongly pseudoconvex CR submanifold: An addendum to the paper of Harvey-Lawson

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The purpose of this paper is to give a counterexample of Theorem 10.4 in [Ha-La]. In the Harvey-Lawson paper, a global result is claimed, but only a local result is proven. This theorem has had a big impact on CR geometry for almost a quarter of a century because one can use the theory of isolated singularities to study the theory of CR manifolds and vice versa.

Example. Consider the following holomorphic map:

$$\begin{array}{rcl} F: \mathbf{C}^2 & \longrightarrow & \mathbf{C}^3 \\ (u,v) & \longrightarrow & (x,y,z) = \Big(u(u-1), v, u^2(u-1)\Big). \end{array}$$

Clearly for any c, F restricted on the line $\{v = c\}$ is an embedding outside the two points $(0,c)$ and $(1,c)$. F sends $(0,t)$ and $(1,t)$ to $(0,t,0)$ for all t. Now take S, which is the boundary of a ball $B = \Big\{(u,v) \in \mathbf{C}^2 : \Big\|(u,v)\Big\| \le 2\Big\}$. It is easy to see that the mapping F restricted on S is still an embedding. The image of S under F is a strongly pseudoconvex CR manifold in $\mathbf{C}^3$. The variety that $F(S)$ bounds is $F(B)$. Observe that $F(B)$ has curve singularities along the line $(0,t,0)$. We remark that $F(\mathbf{C}^2)$ is a hypersurface $\Big\{(x,y,z) \in \mathbf{C}^3 : z^2 - zx - x^3 = 0\Big\}$ in $\mathbf{C}^3$.

Theorem 10.4 of [Ha-La] was so powerful that it has been used by many researchers. Fortunately, we can replace it by the following theorem, the proof of which will appear elsewhere [Lu-Ya].

Theorem. *Let X be a strongly pseudoconvex* CR *manifold of dimension $2n-1$, $n \ge 2$. If X is contained in the boundary of a bounded strictly pseudoconvex domain D in $\mathbf{C}^N$, then there exists a complex analytic subvariety V of dimension n in $D - X$ such that the boundary of V is X. Moreover, V has boundary regularity at every point of X, and V has only isolated singularities in $V|X$.*

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